7.6×10^4, which is comparable with the value of 5×10^4 found by Taylor from wind velocity measurements over Salisbury Plain, where the turbulence would be expected from the nature of the ground to be less than over Paris. The agreement between these values and the value of 10×10^4 from the temperatures observed on Eiffel Tower is quite as good as could be expected, considering the approximations in the calculations, and affords satisfactory confirmation of the theory that momentum and heat are transmitted by the same agency, and that the behavior of the lower atmosphere in transmitting heat can be calculated from observations of the retardation of the lower layers of the earth's atmosphere by the friction of the ground.

The remainder of the paper is devoted to the interpretation of the low-level reversal in the type of diurnal march of wind velocity recently brought to light by Hellmann's observations.[2]

The complementary types of diurnal march of wind velocity (1) with a maximum in the middle of the day, observed near the ground, and (2) with a minimum in the middle of the day, on mountains, are well known. Hellmann's observations with anemometers at 2, 16, and 32 meters above the ground show the upper-air type approaching near enough to the ground when the wind is light to give maxima in the middle of the night at 16 and 32 meters. At 16 meters the maxima of midday and midnight are about equal, at 32 meters the night maximum is greater than the day maximum. In strong wind the march is characterized by a midday maximum, and midnight minimum at all of the anemometers.

The Espy-Köppen theory according to which both types of daily variation are the results of midday convectional ascending currents, the circulation of which carry the more stagnant air up from the ground to reduce the velocity of the higher layers, and the faster-moving upper wind down to the ground to increase the velocity of the surface wind, fails to account for Hellmann's observations, because it leads to the conclusion that the vertical currents due to the heating of the ground must extend to a much greater height in strong winds than they do in light winds.

The theory of turbulence, involving frictional as well as convectional interchange of air at different levels affords a satisfactory quantitative explanation of the phenomena observed by Hellmann. In the absence of determinations of the diurnal variation of K, it is necessary to estimate what this would be from the annual variation of K shown by the Eiffel Tower observations. K is assumed to vary continuously from a maximum at midday to a minimum at midnight, in such a way that the corresponding values of the angle α between the directions of the surface wind and the gradient wind vary through the entirely probable range from 10° at midday to 30° at midnight. The velocities are then taken off a graphic representation of the vertical distribution of wind velocity corresponding to a series of values of α. Curves[3] obtained in this way for a series of arbitrary heights agree exceedingly well with the march actually observed by Dr. Hellmann.

The data obtained in various ways as to the value of K are used to estimate the limits to which the ground type of daily march of wind velocity is likely to extend, the results being given in the following table:

		Gradient velocity *m. p. s.*	At midday.		At midnight.		Height at which maxima at midday and midnight are equal.
			K	α	K	α	
							Meters.
Strong winds	Summer	13.6	40×10^4	12	6×10^3	28	60
	Winter		13×10^4	17			50
Light winds	Summer	4.6	20×10^4	7	1×10^3	26	30
	Winter		7×10^4	10			25

These theoretical conclusions agree with Dr. Hellmann's observations, in which the reversal in light winds occurred at about 16 meters in winter and about 32 meters in summer. The reversal in strong winds was above all three anemometers, and also above the anemometer 41 meters above the ground at the meteorological observatory at Potsdam.

Taylor's coefficient of eddy conductivity appears to be an important meteorological constant. Its applicability in the dynamics of the lower atmosphere is obvious. That it may be of great practical importance is indicated by the successful elucidation of Hellmann's observations, which are evidently closely related to the phenomenon of the nocturnal inversion of temperature. Observations of it may become essential to the successful forecasting of agricultural frosts.

ATMOSPHERIC STIRRING MEASURED BY PRECIPITATION.

By Lewis F. Richardson.[1]

[Abstracted from Proceedings of the Royal Society, Series A, vol. 96, No. 674, pp. 9-18, 1919.]

Gentle mixing of a definite portion of air does not alter the total amount of water in it; any increase in the amount must come from water flowing in over the sides of that portion of air. Taking a large horizontal layer of air, and defining *upward flux* as the ratio of amount of water rising across a large horizontal surface in unit time to the area of the surface, we can *define* a coefficient c, such that

$$\varphi = -c\frac{\partial x}{\partial h},$$

where φ is upward flux, h is height, x is amount of water per unit mass of air. When a definite portion of air is removed from one level to another, the total amount of water associated with it does not, of course, change, and hence x does not change, and φ is zero when $\partial x/\partial h$ is zero. It is then very easy to show that

$$\frac{\partial x}{\partial t}=\frac{\partial}{\partial p}\left(\xi\frac{\partial x}{\partial p}\right)$$

is the equation for diffusion, where p is pressure, and ξ is equal to $g^2\rho c$, ρ being density; also that

$$\varphi = -\frac{\xi}{g^2\rho}\frac{\partial x}{\partial h},$$

ξ being the stirring coefficient, or measure of degree of atmospheric turbulence. Since on the average the water-content of the atmosphere is not increasing, the water which descends as precipitation must have been stirred

[2] Über die Bewegung der Luft in den untersten Schichten der Atmosphäre. Met Zeit, Jan. 1915, vol. 32, pp. 1-16.

[3] Given in the original paper, but not reproduced here.

[1] Cf. Science Abstracts, Nov. 29, 1919, pp. 502-503.

up into the atmosphere. φ gives the mass of water in the form of vapor rising across one horizontal square centimeter per second *on the average of a large horizontal area*, and this must then be equal to the average rate of rainfall at that level. We extend the area to cover the entire globe; then taking the layer of the atmosphere in the kilometer next above the ground, and inserting in the equations the mean values of the vertical gradient of mass of water per mass of air, and the mean values of the density and rate of precipitation, we find ξ at 500 meters to be 140,000 cm.$^{-2}$ grm.^{2} sec.$^{-5}$; similarly, taking the layer in which the upper clouds occur, at which height the precipitation consists only of the slow descent of the clouds themselves, and estimating the amount and size of the cloud-particles (*cf.* L. F. Richardson, *Proc. Roy. Soc.*, A96, 19–31, 1919), it is possible to get ξ at 8.5 kilometers to be from 3 to 180 C. G. S. For the layer in the meter next above the ground, some psychrometric observations indicate that ξ for 0.5 meter is 1,000 or less.

The value at 500 meters is in fair agreement with the value deduced from certain experimental investigations.

In the stratosphere, ξ must, of course, be so small that the equality of temperatures will not be disturbed.

The mean values being taken over the whole globe, the effects of the largest circulatory motions between the poles and the equator are combined with those of rising currents in cyclones, anticyclones, cumulus eddies, etc., in deducing the value of the coefficient of eddy-diffusivity, the latter thus becoming a statistical measure of the effects of circulatory motions which we can not or do not wish to consider in detail.

The quantity x may also be entropy, potential temperature, or velocity in a fixed azimuth, all per unit mass, and hence the equations have applications to the mechanics of eddy motion in the atmosphere.

These equations are an improvement on similar ones derived by Taylor, the effect of altitude being considered (*Phil. Trans.*, A215, p. 3, 1915).—*E. W. W[oolard]*.

A FORMULA FOR THE RELATION OF MEAN WIND VELOCITY TO ALTITUDE WITH RESPECT TO HELLMANN'S INVESTIGATIONS.[1]

By DR. F. BRADTKE.

[Abstracted from *Meteorologische Zeitschrift*, Nov.-Dec., 1918, vol. 35, pp. 313-315.]

In Hellmann's work on "The Motion of Air in the Lower Layers of the Atmosphere," he deduces two formulae to be applied in obtaining the mean wind speed at various elevations, the one for extremely low elevations, the other for the higher. These are:

$$(1)\ v=a+b\log(h+c),\text{ and,}$$

$$(2)\ v/v_0=\sqrt[5]{h/h_0},$$

the first to apply above 123 meters, the second below 16 meters. He implies that there can be no simple expression for use at all altitudes.

The following equation has been determined, which, as will be shown, gives, with great accuracy, the speed at any elevation:

$$v=1.2+1.79\ h^{0.246}$$

[1] Meteorol. Zeitschr., 1917, vol. 34, p. 273.

To show the accuracy with which this formula can be used the following table gives the mean wind velocity as determined at various heights by Hellmann at Nauen; and below them are presented the values as computed by this formula:

$h=$	2	16	32	123	258 meters
$v=$ observed	3.33	4.69	5.40	7.02	8.26 m. p. s.
$v=$ computed	3.32	4.74	5.40	7.05	8.22 m. p. s.

Carrying the test farther, the computed value for 305 meters is 8.51 meters per second, and the observed value on the Eiffel Tower is 8.71 meters per second; the computed speed is 9.32 meters per second for 500 meters and the observed value from Lindenberg kite flights for this elevation is 9.3 meters per second. Hellmann's formula gives ..48 and 9.25 meters per second, respectively.—*C. L. M.*

ON THE DEPENDENCE OF WIND SPEED UPON ALTITUDE.

By V. LASKA.

[Abstracted from *Meteorologische Zeitschrift*, Nov.-Dec., 1918, vol. 35, pp. 315-316.]

The calculations are based upon an interpolation formula in which h is altitude above the ground in meters, and ν the wind speed in meters per second, as follows:

$$\log\nu_x=\log\nu_1\,[1+(x-1)/10] \qquad (1)$$

in which

$$h=2^x,\ h<500\text{ meters.}$$

The comparison of calculated to observed values in the vicinity of the ground is given in the following table:

TABLE 1.

h	2	4	8	16	32	64	128	256	512
ν Calculated	3.4	3.9	4.4	4.9	5.6	6.3	7.1	8.0	9.1
ν Observed	3.3	—	—	4.7	5.4	—	7.0 ($h=$ 123)	8.3 ($h=$ 258)	9.1

When $h=0$, Hellmann has given a value of wind speed of 2.8, but the author takes the stand that the formula gives the correct value of 0.0; in other words, in order to account for the wave-forming tendency of the air, there must be a quiet layer—perhaps only a few millimeters in thickness, next to the ground. It follows that there is a very sudden increase in the immediate vicinity of the ground. The original formula is simplified to—

$$\log\nu/\log h=c \qquad (2)$$

Where there are several elevations we have

$$\log\nu_1\nu_2\ \ldots\ \nu_n/\log h_1h_2\ \ldots\ h_n=c \qquad (3)$$

The demonstration of this is shown in Table 2.

TABLE 2.

	Jersey.	Zikawei.		Nauen.		Strassburg.			Chicago.		
h	55	12	41	16	32	18	52	144	32	47	83
ν	7.4	3.6	5.9	4.7	5.4	2.8	4.2	6.0	4.2	4.6	7.8
c	.050	0.52	0.48	0.56	0.49	0.36	0.36	0.36	0.41	0.40	0.46
c (formula 3)	.050		0.49	0.52		0.36			0.43		